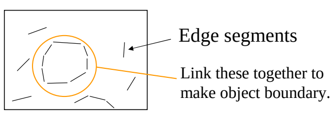
Boundaries often hard to find

Active Contour Models: Snakes

- Important class of algorithms for finding object boundaries
- Usually assume little about shape other than that it is smooth
- Finding boundaries becomes an optimisation process
- Balance between matching to image and ensuring result is smooth

Introduction

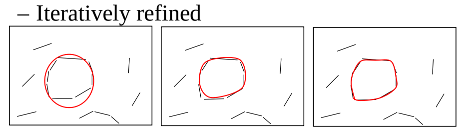
- Suppose we wish to find the boundary of an object in an image
- One approach is to find edge segments and link them together



Edge image

Introduction

- Alternatively use an Active Contour Model (or `snake’)
  - A smooth curve which matches to image data
  - Curve is initialised near target
  - Iteratively refined



Initial position   Intermediate   Final position

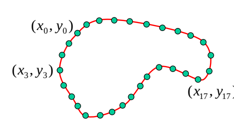
Properties of snakes

- Optimise a combination of *internal* and *external* energy
- Internal energy constrains shape of curve
  - eg to encourage smoothness
- External energy encourages matching to suitable image features
  - Eg strong edges

Simple curve representation

- Represent the curve with a set of *n* points

$$\mathbf{v}_i = (x_i, y_i) \quad i = 0 \ldots n-1$$



Snake Energy

- The total energy of the snake is defined as

$$E_{\text{total}} = E_{\text{internal}} + E_{\text{external}}$$

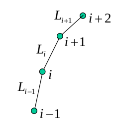
Encourages smoothness or a particular shape

Encourages curve onto image structures (eg edges)

Simple Elastic Curve

- For a curve represented as a set of points a simple elastic energy term is

$$E_{\text{internal}} = \alpha \sum_{i=0}^{n-1} L_i^2$$
$$= \alpha \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2$$



  – This encourages the closed curve to shrink to a point (like a very small elastic band)

External Energy

- The external energy describes how well the curve matches the image data locally
- Numerous forms can be used, attracting the curve toward different image features

Simple Edge Strength

- Suppose we have an image $I(x, y)$
- Generate gradient images $G_x(x, y), G_y(x, y)$
- The absolute edge strength at a point is then

$$| G_x(x, y) |^2 + | G_y(x, y) |^2$$

- An external energy term for a snake is thus

$$E_{external} = -\sum_{i=0}^{n-1} | G_x(x_i, y_i) |^2 + | G_y(x_i, y_i) |^2$$

(Negative in order that minimising it forces the curve toward large edges)

Simple Elastic Snake

- A simple elastic snake is thus defined by
  – a set of *n* points,
  – An internal elastic energy term
  – An external edge based energy term
- To use this to locate the outline of an object
  – Initialise in the vicinity of the object
  – Modify the points to minimise the total energy

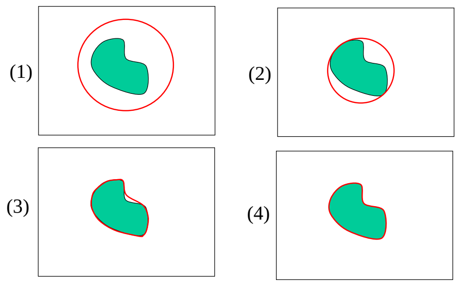
Simple Elastic Snake

- In this case we have

$$E_{\text{total}}(\mathbf{x}) = E_{\text{internal}}(\mathbf{x}) + E_{\text{external}}(\mathbf{x})$$
where $\mathbf{x} = (x_0, \cdots, x_{n-1}, y_0, \cdots, y_{n-1})^T$

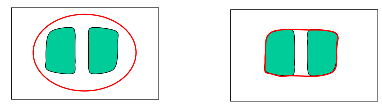
  – which is a 2*n*-D optimisation problem
  – Use a general optimisation algorithm
  – Note that we can easily compute the derivatives, which allows efficient optimisation

Synthetic Example



Dealing with missing data

- The smoothness constraint can deal with missing data:

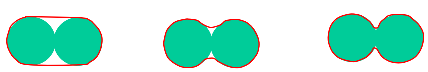


Relative weighting

- Notice that the strength of the internal elastic component can be controlled by a parameter, α

$$E_{\text{internal}} = \alpha \sum_{i=0}^{n-1} L_i^2$$

- Increasing this increases stiffness of curve

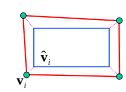


large α medium α small α

Simple shape prior

- If object is some smooth variation on a known shape, use

$$E_{\text{internal}} = \alpha \sum_{i=0}^{n-1} |\mathbf{v}_i - \hat{\mathbf{v}}_i|^2$$



where $\{\hat{\mathbf{v}}_i\}$ give the points of the basic shape

- Perhaps use a proper statistical shape model

Alternative Image Energies

- Directed gradient measures

$$E_{external} = -\sum_{i=0}^{n-1} |u_{x,i} G_x(\mathbf{v}_i) + u_{y,i} G_y(\mathbf{v}_i)|$$

where $(u_{x,i}, u_{y,i})$ is the unit normal to the boundary at the point

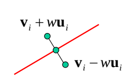
  - This gives a good response when the boundary has the same direction as the edge, but weaker responses when it does not

Alternative Image Energies

- Line strength measure

$$E_{external} = -\sum_{i=0}^{n-1} I(\mathbf{v}_i) - 0.5[I(\mathbf{v}_i + w\mathbf{u}_i) + I(\mathbf{v}_i - w\mathbf{u}_i)]$$

where $\mathbf{u}_i$ is the unit normal at the point, w is estimate of line half - width

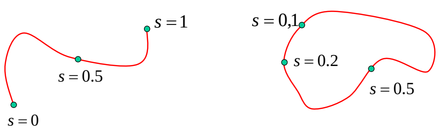


  - Strong response when on a bright line
  - Looks for white lines on dark backgrounds

Continuous Curve Representation

- We can represent the curve parametrically

$$\mathbf{v}(s) = (x(s), y(s)) \quad 0 \le s \le 1$$



Open Curve   Closed Curve

Internal Energy

- The bending energy of a continuous curve is given by

$$E_{\text{int}}(\mathbf{v}(s)) = \underbrace{\alpha(s)\left|\frac{d\mathbf{v}}{ds}\right|^2}_{\text{Elasticity}} + \underbrace{\beta(s)\left|\frac{d^2\mathbf{v}}{ds^2}\right|^2}_{\text{Stiffness}}$$

(The more the curve bends, the larger this value)

Discrete representation

- If the curve is represented by n points

$$\mathbf{v}_i = (x_i, y_i) \quad i = 0 \ldots n-1$$

$$\frac{d\mathbf{v}}{ds} \approx \frac{\mathbf{v}_{i+1} - \mathbf{v}_{i-1}}{2} \qquad \frac{d^2\mathbf{v}}{ds^2} \approx (\mathbf{v}_{i+1} - \mathbf{v}_i) - (\mathbf{v}_i - \mathbf{v}_{i-1}) = \mathbf{v}_{i+1} - 2\mathbf{v}_i + \mathbf{v}_{i-1}$$

$$E_{\text{internal}} = \sum_{i=0}^{n-1} \underbrace{\alpha\,|\mathbf{v}_{i+1} - \mathbf{v}_{i-1}|^2}_{\text{Elasticity}} + \underbrace{\beta\,|\mathbf{v}_{i+1} - 2\mathbf{v}_i + \mathbf{v}_{i-1}|^2}_{\text{Stiffness}}$$

Snake Energy (continuous form)

- The total energy of the snake is defined as

$$E_{\text{total}} = E_{\text{internal}} + E_{\text{external}}$$

$$E_{\text{internal}} = \int_0^1 E_{\text{int}}(\mathbf{v}(s))\,ds$$ Eg Bending energy

$$E_{\text{external}} = \int_0^1 E_{\text{ext}}(\mathbf{v}(s))\,ds$$ Eg. Total edge strength under curve

Open Curves

Closed Curve   *Open Curve*

- When using an open curve we must impose constraints on the end points
- Typically fix them

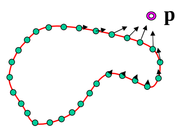
Additional Constraints

- Snakes originally developed for interactive image segmentation
- User could draw initial contour, then nudge it when it goes wrong
- Simply modify the external energy term to
  - Pull nearby points toward cursor, or
  - Push nearby points away from cursor

Interactive forces

- Pull points towards cursor:

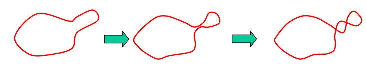
$$E_{pull} = \sum_{i=0}^{n-1} \frac{r^2}{|\mathbf{v}_i - \mathbf{p}|^2}$$



Nearby points get pulled hardest

Problems with snakes

- Just using smoothness doesn’t always capture all useful prior knowledge
- User must define smoothness parameters
- Snake may oversmooth the boundary
- Not trivial to prevent curve self intersecting



Problems with Snakes (II)

- Can’t easily deal with relationships between objects
  - Use a statistical shape model instead
- Choice of most suitable external energy sometimes more an art than a science

Suggested Reading

- “Image Processing, Analysis and Machine Vision” (Sonka,Hlavac and Boyle), pp374-380
  - Gives more mathematical detail and more examples
- For the dedicated
  - “Active Contours”, A.Blake and M.Isard (pub.Springer)

Demos

- Snake demo
  - http://www.isbe.man.ac.uk/~bim/software
- Matlab Snake Demo
  - By Chris Bregler and Malcolm Slaney, Interval Research Corporation.
  - Download from http://www.slaney.org/malcolm/pubs.html